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# CERTAIN REPRESENTATIONS ON SOFT M-N MODULO THEORY

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**ABSTRACT**

In this paper, we define a new concept, called soft intersection action (SI) on M- N module structures on a soft set. This new notions gathers soft set theory and near-ring modulo theory together and it shows how a soft set effects on M- N module structure in the mean of union and inclusion of sets. We then obtain its basic properties with illustrative examples and derive some analog of classical M-N module theoretic concepts for SI-action on M-N-module. Finally , we give the application of SI-actions on M-N module theory.

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**Introduction:** Soft set theory was introduced in 1999 by Molodtsov [22] for dealing with uncertainties and it has gone through remarkably rapid strides in the mean of algebraic structures as in [1, 2, 11, 14, 15, 16, 18, 25, 28]. Moreover, Atagun and Sezgin [4] defined the concepts of soft subrings and ideals of a ring, soft subfields of a field and soft submodules of a module and studied their related properties with respect to soft set operations. Operations of soft sets have been studied by some authors, too. Maji et al. [19] presented some definitions on soft sets and based on the analysis of several operations on soft sets Ali et al. [3] introduced several operations of soft sets and Sezgin and Atagun [26] studied on soft set operations as well. Furthermore, soft set relations and functions [5] and soft mappings [21] with many related concepts were discussed. The theory of soft set has also a wide-ranging applications especially in soft decision making as in the following studies: [6, 7, 23, 29]. In this paper, we define a new concept, called soft intersection action (SI) on M-N module structures on a fuzzy soft set. This new notions gathers fuzzy theory, soft set theory and near-ring modulo theory together and it shows how a soft set effects on M-N module structure in the mean of union and inclusion of sets. We then obtain its basic properties with illustrative examples and derive some analog of classical M-N module theoretic concepts for SI-action on M-N module. Finally, we give the application of SI-actions on M-N module

**2.Preliminaries**: In this section, we recall some basic notions relevant to near-ring modules. By a near-ring, we shall mean an algebraic system (N,+,.), where

$(N_1)$(N, +) forms a group (not necessarily abelian)

$(N_2)$ (N, .) forms a semi group and

$(N_3)$ (x + y)z = xz + yz for all x,y,z ∈ N. (that is we study on right near-ring modules)

Throughout this paper, N will always denote right near-ring. A normal subgroup H of N is called a left ideal of N if n(s+h)-ns ∈ H for all n,s ∈ N and h ∈ I and denoted by $H \lhd_\ell N$. For a near-ring N, the zero-symmetric part of N denoted by $N_0$ is defined by $N_0$={n∈ S / n0=0}.

Let (S,+) be a group and A: N×S →S, (n,s)→s.

(S,A) is called N module or near-ring module if for all x,y ∈ N,for all s ∈ S.

(i) x(ys) = (xy)s

(ii) (x+y)s = xs+ys. It is denoted by $N^S$. Clearly N itself is an N module by natural operations. A subgroup T of $N^S$ with NT⊆T is said to be N-sub module of S and denoted by

$T \le_N S$. A normal subgroup T of S is called an N-ideal of $N^S$ and denoted by a near-ring, S and χ two N-modules. Then h: S→χ is called an M-N-homomorphism if s,$\delta$ ∈ S, for all n∈ N,

(i) h(m(s+$\delta$)) = h(s)+h($\delta$) and

(ii) h(ns) = nh(s).

For all undefined concepts and notions we refer to (24). From now on, U refers to on initial universe, E is a set of parameters P(U) is the power set of U and A,B,C⊆ E.

**2.1.Definition**[22]: A pair (F,A) is called a soft set over U, where F is a mapping given by F : A→P(U).

**2.2.Definition**[6] :The relative complement of the soft set $F_A$ over U is denoted by $F^r_A$, where

$F^r_A$: A → P(U) is a mapping given as $F^r_A(a)$ =U \\$F_A(a)$, for all a ∈A.

**2.3.Definition**[6]: Let $F_A$ and $G_B$ be two soft sets over U such that A∩B ≠ ∅,. The restricted intersection of $F_A$ and $G_B$ is denoted by $F_A⋓G_B$, and is defined as $F_A⋓G_B$ =(H,C), where

C = A∩B and for all c ∈C, H(c) = F(c)∩G(c).

**2.4.Definition**[6]: Let $F_A$ and $G_B$ be two soft sets over U such that A∩B ≠ ∅,. The restricted union of $F_A$ and $G_B$ is denoted by $F_A \cup_R G_B$, and is defined as $F_A \cup_R G_B$ = (H,C),where C = A∩B and for all c ∈C, H(c) = F(c)∪G(c).

**2.5Definition**[12]: Let $F_A$ and $G_B$ be soft sets over the common universe U and $\psi$be a function from A to B. Then we can define the soft set $\psi(F_A)$ over U, where$\psi$ $(F_A)$ : B→P(U) is a set valued function defined by$\psi$ $(F_A)(b)$ =∪{F(a) | a ∈A and$\psi$ (a) = b},

if$\psi^{-1}(b) \neq \emptyset$, = 0otherwise for all b ∈B. Here, $\psi$ $(F_A)$ is called the soft image of $F_A$ under $\psi$. Moreover we can define a soft set $\psi^{-1}(G_B)$ over U, where $\psi^{-1}(G_B)$ : A → P(U) is a set-valued function defined by $\psi^{-1}(G_B)(a)$ = G($\psi$ (a)) for all a ∈A. Then, $\psi^{-1}(G_B)$ is called the soft pre image (or inverse image) of $G_B$ under $\psi$.

**2.6.Definition**[13]: Let $F_A$ and $G_B$ be soft sets over the common universe U and $\psi$be a function from A to B. Then we can define the soft set $\psi^\star(F_A)$ over U, where $\psi^\star(F_A)$ : B→P(U) is a set-valued function defined by $\psi^\star(F_A)(b)$=∩{F(a) | a ∈A and$\psi$ (a) = b}, if $\psi^{-1}(b) \neq \emptyset$,

=0 otherwise for all b ∈B. Here, $\psi^\star(F_A)$ is called the soft anti image of $F_A$ under $\psi$.

**2.7 Definition** [8]:Let $f_A$be a soft set over U and α be a subset of U. Then, upper α-inclusion of a soft set $f_A$, denoted by $f^{\alpha}A$, is defined as $f^{\alpha}A = \{x \in A : f_A(x) \supseteq \alpha\}$

**3.SI-action on M-N module structures and M-N-ideal structures**

In this section, we first define fuzzy soft union action, abbreviated as SI-action on M-N module and M-N-ideal structures with illustrative examples. We then study their basic results with respect to soft set operation.

**3.1 Definition:** Let S be an M-N module and $f_s$ be a soft set over U.Then$f_s$ is called SI-action on M-N module over U if it satisfies the following conditions;

($S_I$M-1) $f_s(m(x+y)) \supseteq f_s(x) \cap f_s(y)$

($S_I$M-2) $f_s(-x) \supseteq f_s(x)$

($S_I$M-3) $f_s(nx) \supseteq f_s(x)$

For all x, y ∈ S and n ∈N m ∈ M .

**3.2 Example:** Consider the module M-N ={0,x,y,z},be the near-ring under the operation defined by the following table:

| + | 0 | x | y | z |
|---|---|---|---|---|
| 0 | 0 | x | y | z |
| x | x | 0 | z | y |
| y | y | z | 0 | x |
| z | z | y | x | 0 |

| . | 0 | x | y | z |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| x | x | x | x | x |
| y | 0 | 0 | 0 | 0 |
| z | x | x | x | x |

Let S=M-N and S be the set of parameters

and U=$\left\{\begin{bmatrix} a & a \\ 0 & a \end{bmatrix} / a, b \in Z_6\right\}$,2×2 matrices with $Z_6$ terms, is the universal set .we construct a fuzzy soft set.

$f_s(0)= \left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 3 \end{bmatrix}\right\}$,$f_s$ (x)= $\left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 3 \end{bmatrix}\right\}$,

$f_s$ (y)= $\left\{\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}\right\}$, and $f_s$ (z)= $\left\{\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}\right\}$

Then one can easily show that the soft set $f_s$ is a SI-action on M-N module.

**3.3 Proposition:** Let $f_s$ be a SI-action on M-N module over U. Then, $f_s(0) \supseteq f_s(x)$ for all x∈ $S$.

**Proof:** Assume that $f_s$ is SI-action over U.Then, for all x∈S ,

$f_s(m0)= f_s(m(x-x)) \supseteq f_s(x) \cap f_s(-x)=f_s(x) \cap f_s(x)=f_s(x)$.

**3.4 Theorem:** Let S be a SI-action on M-N module and $f_s$ be a soft set over U.

Then$f_s$ is SI-action of M-N module over U if and only if

(i)$f_s(m(x-y)) \supseteq f_s(x) \cap f_s(y)$

(ii) $f_S(nx) \supseteq f_S(x)$ for all x,y ∈ S and n ∈ N.

**Proof:** Suppose $f_S$ is a fuzzy SI-action on M-N module over U. Then, by definition-3.1,

$f_S(xy) \supseteq f_S(y)$ and $f_S(m(x-y)) \supseteq f_S(x) \cap f_S(-y) = f_S(x) \cap f_S(y)$ for all x,y ∈ S

Conversely, assume that $f_S(xy) \supseteq f_S(y)$ and $f_S(m(x-y)) \supseteq f_S(x) \cap f_S(y)$ for all x,y ∈ S.

If we choose x=0,then $f_S(0-y)= f_S(-y) \supseteq f_S(0) \cap f_S(y) = f_S(y)$ by proposition-3.1. Similarly $f_S(ny)= f_S(-(-y)) \supseteq f_S(-y)$,thus $f_S(-y) = f_S(y)$ for all y ∈ S. Also ,by assumption

$f_S(m(x-y)) \supseteq f_S(x) \cap f_S(-y) = f_S(x) \cap f_S(y)$. This complete the proof.

**3.5 Theorem:** Let $f_S$ be a SI-action on M-N module over U.

(i)If $f_S(m(x-y))= f_S(0)$ for any x,y ∈ S ,then $f_S(x) = f_S(y)$.

(ii) $f_S(m(x-y))= f_S(0)$ for any x, y ∈ S, then $f_S(x) = f_S(y)$.

**Proof:** Assume that $f_S(x-y)= f_S(0)$ for any x,y ∈ S ,then

$f_S(mx) = f_S(m(x-y+y)) \supseteq f_S(x-y) \cap f_S(y)$

$= f_S(0) \cap f_S(y) = f_S(y)$

and similarly,

$f_S(ny) = f_S(n(y-x)+x)) \supseteq f_S(y-x) \cap f_S(x)$

$= f_S(-(y-x)) \cap f_S(x)$

$= f_S(0) \cap f_S(x) = f_S(x)$

Thus, $f_S(x) = f_S(y)$ which completes the proof .Similarly, we can show the result (ii).

It is known that if S is M-N module, then (S, +)is a group but not necessarily abelian. That is, for any x, y ∈ S, x + y needs not be equal to y + x. However, we have the following:

**3.6 Theorem :** Let $f_S$ be SI-action on M-N module over U and x ∈ S. Then,

$f_S(x) = f_S(0) \Leftrightarrow f_S(x+y) = f_S(y+x) = f_S(y)$ for all y ∈ S .

**Proof:** Suppose that $f_S(x+y)= f_S(y+x) = f_S(y)$ for all y ∈ S. Then, by choosing y=0,

we obtain that $f_S(x) = f_S(0)$.

Conversely, assume that $f_S(x) = f_S(0)$. Then by proposition-3.1,we have

$f_S(0) = f_S(x) \supseteq f_S(y), \forall$ y ∈ S…………… (1)

Since $f_S$ SI-action on M-N module over U, then

$f_S(m(x + y)) \supseteq f_S(x) \cap f_S(y) = f_S(y), \forall$ y ∈ S. Moreover, for all y ∈ S

$f_S(ny) = f_S(n(-x)+x)+y)) = f_S(n(-x+(x + y))) \supseteq f_S(-x) \cap f_S(x+y)$

$= f_S(x) \cap f_S(x+y)= f_S(x+y)$

Since by equation (1), $f_S(x) \supseteq f_S(y)$ for all y ∈ S and x, y ∈ S ,implies that x + y ∈ S .Thus, it follows that $f_S(x) \supseteq f_S(x+y)$. So $f_S(x+y) = f_S(y)$ for all y ∈ S.

Now, let x ∈ S. Then, for all x, y ∈ S

$f_S(m(y + x)) = f_S(m(y+x+(y-y)))$

$= f_S(m(y+(x+y)-y))$

$\supseteq f_S(y) \cap f_S(x+y) \cap f_S(y)$

$= f_S(y) \cap f_S(x+y)= f_S(y)$

Since $f_S(x+y) = f_S(y)$.Furthermore, for all y ∈ S

$f_S$(ny) $= f_S(n$(y+(x-x)))
$= f_S$((y + x)-x )
$\supseteq f_S$(y + x)$\cap f_S$(x)
$= f_S$(y + x) by equation(1).

It follows that $f_S$(y+x) = $f_S$(y) and so $f_S$(x+y)= $f_S$(y+x)= $f_S$(y) , for all y $\in$ S ,which completes the proof.

**3.7 Theorem:** Let S be a near-field and $f_S$ be a soft set over U. If $f_S(0) \supseteq f_S(1) = f_S(x)$ for all $0 \neq x \in S$, then it is SI-action on M-N module over U.

**Proof:** Suppose that $f_S(0) \supseteq f_S(1) = f_S(x)$ for all $0 \neq x \in S$. In order to prove that it is SI-action on M-N module over U,it is enough to prove that $f_S(m(x-y)) \supseteq f_S(x) \cap f_S(y)$ and $f_S(nx) \supseteq f_S(x)$.

Let x, y $\in$ S. Then we have the following cases:

**Case-1:** Suppose that x$\neq$ 0 and y=0 or x=0 and y$\neq$ 0. Since S is a near-field, so it follows that nx=0and $f_S(nx) = f_S(0)$.since $f_S(0) \supseteq f_S(x)$,for all x $\in$ S ,so $f_S(nx) = f_S(0) \supseteq f_S(x)$,and $f_S(nx) = f_S(0) \supseteq f_S(y)$. This imply $f_S(nx) \supseteq f_S(x)$.

**Case-2:** Suppose that x$\neq$ 0 and y$\neq$0. It follows that nx$\neq$0.Then, $f_S(nx) = f_S(1) = f_S(x)$and $f_S(nx) = f_S(1) = f_S(y)$,which implies that $f_S(nx) \supseteq f_S(x)$.

**Case-3:** suppose that x= 0 and y=0, then clearly $f_S(nx) \supseteq f_S(x)$.Hence $f_S(nx) \supseteq f_S(x)$, for all x, y $\in$ S.

Now, let x, y $\in$ S. Then x-y=0 or x + y$\neq$ *0*.If x + y=0,then either x=y=0 or x$\neq$ *0*,y$\neq$ *0* and x=y.

But, since $f_S(x+y) = f_S(0) \supseteq f_S(x)$,for all x $\in$ N, it follows that $f_S(m(x+y)) = f_S(m0) \supseteq f_S(x) \cap f_S(y)$.

If x + y$\neq$0,then either x$\neq$ *0*,y$\neq$ *0* and x$\neq$y or x$\neq$ *0* and y=0 or x=0 and y$\neq$ *0*.

Assume that x$\neq$ *0*,y$\neq$ *0* and x$\neq$y.This follows that

$f_S(m(x-y)) = f_S(1) = f_S(x) \supseteq f_S(x) \cap f_S(y)$.

Now, let x$\neq$ *0* and y=0. Then $f_S(m(x + y)) \supseteq f_S(x) \cap f_S(y)$. Finally, let x=0 and y$\neq$ *0*.

Then, $f_S(m(x + y)) \supseteq f_S(x) \cap f_S(y)$. Hence $f_S(x-y) \supseteq f_S(x) \cap f_S(y)$, for all x, y $\in$ S.

Thus, $f_S$ is SI-action on M-N module over U.

**3.8 Theorem:** Let $f_S$ and $f_T$ be two SI-action on M-N module over U. Then $f_S \wedge f_T$ is soft SI-action on M-N module over U.

**Proof:** let $(x_1, y_1), (x_2, y_2) \in$ S×T.Then

$$
\begin{aligned}
f_{S\wedge T}\big(m((x_1,y_1) - (x_2,y_2))\big) &= f_{S\wedge T}(m(x_1 - x_2, y_1 - y_2)) \\
&= f_S(x_1 + x_2) \cap f_T(y_1 + y_2) \\
&\supseteq (f_S(x_1) \cap f_S(x_2)) \cap (f_T(y_1) \cap f_T(y_2)) \\
&= (f_S(x_1) \cap f_T(y_1)) \cap (f_S(x_2) \cap f_T(y_2)) \\
&= f_{S\wedge T}(x_1,y_1) \cap f_{S\wedge T}(x_2,y_2)
\end{aligned}
$$

and

$$f_{S\wedge T}\left((n_1,n_2),(x_2,y_2)\right) = f_{S\wedge T}(n_1x_2,n_2y_2)$$
$$= f_S(n_1x_2)\cap f_T(n_2y_2)$$
$$\supseteq f_S(x_2)\cap f_T(y_2)$$
$$= f_{S\wedge T}(x_2,y_2)$$

Thus $f_S \wedge f_T$ is SI-action on M-N module over U.

Note that $f_S \vee f_T$ is not SI-action on M-N module over U.

**3.9 Example:** Assume U $=p_3$ is the universal set. Let S = $Z_3$ and H $=\left\{\begin{bmatrix} a & a\\ b & b\end{bmatrix} / a,b \in Z_3\right\}$ $2\times 2$ matrices with $Z_3$ terms, be set of parameters. We define SI-action on M-N module $f_S$ over U=$p_3$ by

$f_S(0) = p_3$

$f_S(1) = \{(1),(1\ 2),(1\ 3\ 2)\}$

$f_S(2) = \{(1),(1\ 2),(1\ 2\ 3),(1\ 3\ 2)\}$

We define SI-action on M-N-module $f_H$ over U=$p_3$ by

$f_H\left\{\begin{bmatrix} 0 & 0\\ 0 & 0\end{bmatrix}\right\} = p_3$

$f_H\left\{\begin{bmatrix} 0 & 0\\ 1 & 1\end{bmatrix}\right\} = \{(1),(1\ 2),(1\ 3\ 2)\}$

Then $f_S \vee f_T$ is not SI-action on M-N module over U.

**3.10 Definition :** Let $f_S, g_T$ be SI-action on M-N module over U. Then product of fuzzy SI-action on M-N module $f_S$ and $g_T$ is defined as $f_S \times g_T = h_{S\times T}$, where $h_{S\times T}(x,y) = f_S(x)\times g_T(y)$ for all $(x,y)\in S\times T$.

**3.11 Theorem :** If $f_S$ *and* $g_T$ are SI-action on M-N module over U. Then so is $f_S \times g_T$ over U×U.

**Proof:** By definition-3.2, let $f_S \times g_T = h_{S\times T}$, where $h_{S\times T}(x,y) = f_S(x)\times g_T(y)$ for all $(x,y)\in S\times T$. Then for all $(x_1,y_1),(x_2,y_2)\in S\times T$ and $(n_1,n_2)$=N×N.

$$h_{S\times T}\big(m((x_1,y_1)-(x_2,y_2))\big) = h_{S\times T}\,(m(x_1+x_2,y_1+y_2))$$
$$= f_S\,(m(x_1+x_2,\times g_T\,(m(y_1+y_2)))$$
$$\supseteq (f_S(x_1)\cap f_S(x_2))\times(g_T(y_1)\cap g_T(y_2))$$
$$=(f_S(x_1)\times g_T(y_1))\cap(f_S(x_2)\times g_T(y_2))$$
$$= h_{S\times T}(x_1,y_1) - h_{S\times T}(x_2,y_2)$$

$$h_{S\times T}\left((n_1,n_2)(x_2,y_2)\right) = h_{S\times T}(n_1x_2,n_2y_2)$$
$$= fs\,(n_1x_2)\times g_T(n_2y_2)$$
$$\supseteq f_S(x_2)\times g_T(y_2)$$
$$= h_{S\times T}(x_2,y_2)$$

Hence $f_S \times g_T = h_{S\times T}$ is SI-action on M-N module over U.

**3.12Theorem:** If $f_S$ and $h_S$ are SI-action on M-N module over U, then so is $f_S \tilde{\cap} h_S$ over U.

**Proof:** Let $x, y\in$ s and n $\in$ N then

$(f_S \tilde{\cap} h_S)(m(x+y)) = f_S(m(x+y)) \cap h_S(m(x+y))$

$\supseteq (f_S(x) \cap f_S(y)) \cap (h_S(x) \cap h_S(y))$

$= (f_S(x) \cap h_S(x)) \cup (f_S(y) \cap h_S(y))$

$= (f_S \tilde{\cap} h_S)(x) \cap (f_S \tilde{\cap} h_S)(y)$

$(f_S \tilde{\cap} h_S)(nx) = f_S(nx) \cap h_S(nx)$

$\supseteq f_S(x) \cap h_S(x) = (f_S \tilde{\cap} h_S)(x)$. Therefore, $(f_S \tilde{\cap} h_S)$ is SI-action on N-module over U.

**4.SI-action on M-N-ideal structures**

**4.1 Definition:** Let S be an M-N module and $f_S$ be a soft set over U. Then $f_S$ is called SI-action on M-N-ideal of S over U if the following conditions are satisfied:

(i) $f_S(m(x+y)) \supseteq f_S(x) \cap f_S(y)$

(ii) $f_S(-x) = f_S(x)$

(iii) $f_S(x+y-x) \supseteq f_S(y)$

(iv) $f_S(n(x+y)-nx) \supseteq f_S(y)$ for all $x, y \in S$ and $n \in N$.

Here, note that

$f_S(x+y) \supseteq f_S(x) \cap f_S(y)$ and $f_S(-x) = f_S(x)$ imply $f_S(x-y) \supseteq f_S(x) \cap f_S(y)$

**4.2 Example:** Consider the near –ring N={0, x, y, z} with the following tables

| + | 0 | x | y | z |
|---|---|---|---|---|
| 0 | 0 | x | y | z |
| x | x | 0 | z | y |
| y | y | z | 0 | x |
| z | z | y | x | 0 |

| . | 0 | x | y | z |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| x | 0 | 0 | 0 | x |
| y | 0 | x | y | y |
| z | 0 | x | y | z |

Let S=N be the parameters and U=$D_2$,dihedral group, be the universal set. We define a fuzzy soft set $f_S$ over U by $f_S(0) = D_2$, $f_S(x) = \{e, b, ba\}$, $f_S(y) = \{a, b\}$, $f_S(z) = \{b\}$.

Then, one can show that $f_S$ is SI-action on M-N-ideal of S over U.

**4.3 Example:** Consider the near –ring N={0,1,2,3} with the following tables

| + | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 |
| 1 | 1 | 2 | 3 | 0 |
| 2 | 2 | 3 | 0 | 1 |
| 3 | 3 | 0 | 1 | 2 |

| . | 0 | x | y | z |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| x | 0 | 1 | 0 | 1 |
| y | 0 | 3 | 0 | 3 |
| z | 0 | 2 | 0 | 2 |

Let S=N be the set of parameters and U= $Z^+$ be the universal set. We define a fuzzy soft set $f_S$ over U by

$$f_S(0) = \{1,2,3,5,6,7,9,10,11,17\}$$

$$f_S(1) = f_S(3) = \{1,3,5,7,9,11\}$$

$$f_S(2) = \{1,5,7,9,11\}$$

Since $f_S(2.(3+1) - 2.3) = f_S(2.1 - 2.3) = f_S(3-3) = f_S(0) \nsubseteq f_S(1)$

Therefore, $f_S$ is not SI-action on M-N-ideal over U.

It is known that if N is a zero- symmetric near-ring, then every M-N-ideal of S is also M-N module of S. Here, we have an analog for this case.

**4.4 Theorem :** Let N be a zero- symmetric near-ring. Then, every SI-action on M-N-ideal is SI-action on M-N module over U.

**Proof:** Let $f_S$ be an SI-action on M-N-ideal on S over U. Since $f_S$ (n(x + y)- nx) $\supseteq f_S$ (y), for all x, y $\in$ S, and n$\in$N, in particular for x=0, it follows that $f_S$ (n(0+y)-n.0) = $f_S$ (ny-0)= $f_S$ (y) $\supseteq f_S$ (y). Since the other condition is satisfied by definition-4.1, $f_S$ is SI-action on M-N-ideals of S over U.

**4.5 Theorem :** Let $f_S$ be SI-action on M-N-ideal of S and $f_T$ be SI-action on M-N-ideal of T over U. Then $f_S \wedge f_T$ is SI-action on M-N-ideal of S×T over U.

**4.6 Theorem :** If $f_S$ is SI-action on M-N-ideal of S and $f_T$ be SI-action on M-N-ideal of T over U, then $f_S \times f_T$ is SI-action on M-N-ideal over U×U.

**4.7 Theorem :** If $f_S$ and $h_S$ are two SI-action on M-N modules of S over U, then $f_S \tilde{\cap} h_S$ is SI-action on M-N-ideal over U.

**5.Application of SI-action on M-N-module**

In this section, we give the applications of soft image, soft pre-image, lower $\alpha$-inclusion of soft sets and N-module homomorphism with respect to SI-action on M-N module and M-N-ideals.

**5.1Theorem :** If $f_S$ is SI-action on M-N-ideal of S over U, then $S^f$={x $\in$S / $f_S$ (x)= $f_S$ (0)} is a M-N-ideal of S.

**Proof:** It is obvious that 0 $\in S^f$ we need to show that (i) x-y $\in S^f$, (ii) s+x-s $\in S^f$ and (iii) n(s + x)-ns $\in S^f$ for all x,y $\in S^f$ and n$\in$N and s$\in$S.

If x, y$\in S^f$, then $f_S$ (x)= $f_S$ (y) = $f_S$ (0). By proposition-3.1,

$f_S$ (0) $\supseteq f_S$ (x-y), $f_S$ (0) $\supseteq f_S$ (s+x-s), and $f_S$ (0) $\supseteq f_S$ (n(s+x)-ns) for all x,y $\in S^f$ and n$\in$N and s$\in$S.

Since $f_S$ is SI-action on M-N-ideal of S over U, then for all x, y $\in S^f$ and n $\in$ N and s$\in$S.

(i) $f_S$ (m(x-y)) $\supseteq f_S$ (x) $\cap$ $f_S$ (y) = $f_S$ (0).

(ii) $f_S$ (s+x-s) $\supseteq f_S$ (x) = $f_S$ (0).

(iii) $f_S$ (n(s+x)-ns) $\supseteq f_S$ (x) = $f_S$ (0).

Hence $f_S$ (x-y) = $f_S$ (0), $f_S$ (s+x-s)= $f_S$ (0) and $f_S$ (n(s+x)-ns) = $f_S$ (0),for all x,y $\in S^f$ and n$\in$N and s$\in$S.

Therefore $S^f$ is M-N-ideal of S.

**5.2 Theorem :** Let $f_S$ be soft set over U and $\alpha$ be a subset of U such that $\emptyset \supseteq \alpha \supseteq f_S$ (0). If $f_S$ is SI-action on M-N-ideal over U, then $f_S^{\supseteq\alpha}$ is an N-ideal of S.

**Proof:** Since $f_S$ (0) $\supseteq \alpha$, then 0 $\in f_S^{\supseteq\alpha}$ and $\emptyset \neq f_S^{\subseteq\alpha} \supseteq$ S. Let x,y $\in f_S^{\supseteq\alpha}$, then $f_S$ (x) $\supseteq \alpha$ and $f_S$ (y) $\supseteq \alpha$. We need to show that

(i) $x-y \in f_S^{\supseteq\alpha}$

(ii) $s + x- s \in f_S^{\supseteq\alpha}$

(iii) $n(s + x)- ns \in f_S^{\supseteq\alpha}$ for all $x,y \in f_S^{\supseteq\alpha}$ and $n\in N$ and $s\in S$.

Since $f_S$ is SI-action on M-N-ideal over U, it follows that

(i) $f_S(m(x-y)) \supseteq f_S(x) \cap f_S(y) \supseteq \alpha \cap \alpha = \alpha$,

(ii) $f_S(s+x-s) \supseteq f_S(x) \supseteq \alpha$ and

(iii) $f_S(n(s+x)-ns) \supseteq f_S(x) \supseteq \alpha$. Thus, the proof is completed.

**5.3 Theorem :** Let $f_S$ and $f_T$ be soft sets over U and $\chi$ be an M-N-isomorphism from S to T.If $f_S$ is SI-action on M-N-ideal of S over U, then $\chi(f_S)$ is SI-action on M-N-ideal of T over U.

**Proof:** Let $\delta_1,\delta_2$ and $n\in N$. Since $\chi$ is surjective, there exists $s_1,s_2 \in S$ such that $\chi(s_1) = \delta_1$ and $\chi(s_2)=\delta_2$. Then

$$(\chi f_S)(m(\delta_1-\delta_2)) = \cup \{ f_S(s) / s\in S , \chi(s) = \delta_1-\delta_2 \}$$
$$= \cup \{ f_S(s) / s\in S , s =\chi^{-1}(\delta_1-\delta_2) \}$$
$$= \cup \{ f_S(s) / s\in S , s = \chi^{-1}(\chi(s_1-s_2))= s_1-s_2\}$$
$$= \cup \{ f_S(s_1-s_2) / s_i \in S , \chi(s_i) = \delta_i , i =1,2,\ldots\}$$
$$\supseteq \cup \{ f_S(s_1) \cap f_S(s_2)) / s_i \in S , \chi(s_i) = \delta_i , i =1,2,\ldots\}$$
$$=(\{\cup \{f_S(s_1)/s_1 \in S , \chi(s_1) = \delta_1\}\} \cap \{\cup \{f_S(s_2)/s_2 \in S , \chi(s_2) = \delta_2\}\})$$
$$= (\chi(f_S))(\delta_1) \cap (\chi(f_S))(\delta_2)$$

Also $(\chi f_S)(\delta_1+\delta_2-\delta_1) = \cup \{ f_S(s) / s\in S , \chi(s) = \delta_1+\delta_2-\delta_1 \}$

$$= \cup \{ f_S(s) / s\in S , s = \chi^{-1}(\delta_1+\delta_2-\delta_1) \}$$
$$= \cup \{ f_S(s) / s\in S , s = \chi^{-1}(\chi(s_1+s_2-s_1))= s_1+s_2-s_1\}$$
$$= \cup \{ f_S(s_1+s_2-s_1) / s_i \in S , \chi(s_i) = \delta_i , i =1,2,\ldots\}$$
$$\supseteq \cup \{f_S(s_2)/s_2 \in S , \chi(s_2) = \delta_2\}$$
$$= (\chi(f_S))(\delta_2)$$

Furthermore, $(\chi f_S)(n(\delta_1+\delta_2)-n\delta_1) = \cup \{ f_S(s) / s\in S , \chi(s) = n(\delta_1+\delta_2)-n\delta_1 \}$

$$= \cup \{ f_S(s) / s\in S , s = \chi^{-1}(n(\delta_1+\delta_2)-n\delta_1) \}$$
$$= \cup \{ f_S(s) / s\in S , s = n(s_1+s_2)-ns_1\}$$
$$= \cup \{ f_S(n(s_1+s_2)-ns_1) / s_i \in S , \chi(s_i) = \delta_i , i =1,2,\ldots\}$$
$$\supseteq \cup \{f_S(s_2)/s_2 \in S , \chi(s_2) = \delta_2\}$$
$$= (\chi(f_S))(\delta_2).$$

Hence $\chi(f_S)$ is SI-action on M-N-ideal of T over U.

**5.4 Theorem :** Let $f_S$ and $f_T$ be soft sets over U and $\chi$ be an M-N-isomorphism from S to T. If $f_T$ is SI-action on M-N-ideal of T over U, then $\chi^{-1}(f_T)$ is SI-action on M-N-ideal of S over U.

**Proof:** Let $s_1,s_2 \in S$ and $n\in N$. Then

$$(\chi^{-1}(f_T))(m(s_1-s_2)) = f_T(\chi(s_1-s_2))$$
$$= f_T(\chi(s_1)-\chi(s_2))$$
$$\supseteq f_T(\chi(s_1)) \cap f_T(\chi(s_2))$$

$$= (\chi^{-1}(f_T))(s_1) \cup (\chi^{-1}(f_T))(s_2).$$

Also $(\chi^{-1}(f_T))\,(s_1 + s_2 - s_1) = f_T(\chi\,(s_1 + s_2 - s_1))$

$$= f_T(\chi\,(s_1) + \chi(s_2) - \chi(s_1))$$

$$\supseteq f_T(\chi\,(s_2)) = (\chi^{-1}(f_T))(s_2)$$

Furthermore, $(\chi^{-1}(f_T))\,(\mathrm{n}(s_1 + s_2) - \mathrm{n}s_1) = f_T(\chi\,(\mathrm{n}(s_1 + s_2) - \mathrm{n}s_1))$

$$= f_T(\mathrm{n}(\chi\,(s_1) + \chi(s_2)) - \mathrm{n}\chi(s_1))$$

$$\supseteq f_T(\chi\,(s_2)) = (\chi^{-1}(f_T))(s_2)$$

Hence, $(\chi^{-1}(f_T))$ is SI-action on M-N-ideal of S over U.

**Conclusion:** we have defined a new type of M-N-module action on a soft set, called SI-action on M-N-module by using the soft sets. This new concept picks up the soft set theory and M-N-module theory together and therefore, it is very functional for obtaining results in the mean of M-N-module structure. Based on this definition, we have introduced the concept of SI-action on M-N-ideal. We have investigated these notions with respect to soft image, soft pre-image and upper$\alpha$-inclusion of soft sets. Finally, we give some application of SI-action on M-N-ideal to M-N-module theory.

**Future Work**: To extend this study, one can further study the other algebraic structures such as different algebra's like KU-ideals and R- ideals in view of their SI-actions.

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